Cambios de coordenadas

$$x = r \sin\theta \cos\phi$$
$$y = r \sin\theta \sin\phi$$
$$z = r \cos\theta$$

Cartesianas a esféricas

$$\hat{r} = \sin\theta \cos\phi \hat{x} + \sin\theta \sin\phi \hat{y} + \cos\theta \hat{z}$$
$$\hat{\theta} = \cos\theta \cos\phi \hat{x} + \cos\theta \sin\phi \hat{y} - \sin\theta \hat{z}$$
$$\hat{\phi} = -\sin\phi \hat{x} + \cos\phi \hat{y}$$

Esféricas a cartesianas

$$\hat{x} = \sin\theta \cos\phi \hat{r} + \cos\theta \cos\phi \hat{\theta} - \sin\phi \hat{\phi}$$
$$\hat{y} = \sin\theta \sin\phi \hat{r} + \cos\theta \sin\phi \hat{\theta} + \cos\phi \hat{\phi}$$
$$\hat{z} = \cos\theta \hat{r} - \sin\theta \hat{\theta}$$

Identidades vectoriales

$$\nabla \cdot (\nabla \times \mathbf{A}) = 0$$
$$\nabla \times \nabla \psi = 0$$
$$\nabla(\phi + \psi) = \nabla \phi + \nabla \psi$$
$$\nabla(\phi\psi) = \phi\nabla \psi + \psi\nabla \phi$$
$$\nabla \cdot (\mathbf{A} + \mathbf{B}) = \nabla \cdot \mathbf{A} + \nabla \cdot \mathbf{B}$$
$$\nabla \times (\mathbf{A} + \mathbf{B}) = \nabla \times \mathbf{A} + \nabla \times \mathbf{B}$$
$$\nabla \cdot (\psi \mathbf{A}) = \mathbf{A} \cdot \nabla \psi + \psi\nabla \cdot \mathbf{A}$$
$$\nabla \times (\psi \mathbf{A}) = \nabla \psi \times \mathbf{A} + \psi\nabla \times \mathbf{A}$$
$$\nabla(\mathbf{A} \cdot \mathbf{B}) = (\mathbf{A} \cdot \nabla)\mathbf{B} + (\mathbf{B} \cdot \nabla)\mathbf{A} + \mathbf{A} \times (\nabla \times \mathbf{B}) + \mathbf{B} \times (\nabla \times \mathbf{A})$$
$$\nabla \cdot (\mathbf{A} \times \mathbf{B}) = \mathbf{B} \cdot \nabla \times \mathbf{A} - \mathbf{A} \cdot \nabla \times \mathbf{B}$$
$$\nabla \times (\mathbf{A} \times \mathbf{B}) = \mathbf{A}\nabla \cdot \mathbf{B} - \mathbf{B}\nabla \cdot \mathbf{A} + (\mathbf{B} \cdot \nabla)\mathbf{A} - (\mathbf{A} \cdot \nabla)\mathbf{B}$$
$$\nabla \times \nabla \times \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A}$$

Teoremas vectoriales

$$\oint_c \mathbf{A} \cdot d\mathbf{l} = \iint_s (\nabla \times \mathbf{A}) \cdot d\mathbf{s}$$
$$\oiint_s \mathbf{A} \cdot d\mathbf{s} = \iiint_v (\nabla \cdot \mathbf{A}) dv$$
$$\oiint_s (\hat{n} \times \mathbf{A}) ds = \iiint_v (\nabla \times \mathbf{A}) dv$$
$$\oiint_s \psi d\mathbf{s} = \iiint_v \nabla \psi dv$$
$$\oint_c \psi d\mathbf{l} = \iint_s \hat{n} \times \nabla \psi ds$$

Operadores vectoriales

Coordenadas cartesianas

$$\nabla \psi = \hat{x}\frac{\partial \psi}{\partial x} + \hat{y}\frac{\partial \psi}{\partial y} + \hat{z}\frac{\partial \psi}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{x}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{y}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{z}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right) = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla^2 \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \hat{x}\nabla^2 A_x + \hat{y}\nabla^2 A_y + \hat{z}\nabla^2 A_z$$

Operadores vectoriales

Coordenadas cilíndricas

$$\nabla \psi = \hat{\rho}\frac{\partial \psi}{\partial \rho} + \hat{\phi}\frac{\partial \psi}{\partial \phi} + \hat{z}\frac{\partial \psi}{\partial z}$$

$$\nabla \cdot \mathbf{A} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\rho A_\rho + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \mathbf{A} = \hat{\rho}\left(\frac{1}{\rho}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\phi}\left(\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho}\right) + \hat{z}\frac{1}{\rho}\left(\frac{\partial}{\partial \rho} r A_\phi - \frac{\partial A_\rho}{\partial \phi}\right)$$

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\rho}\frac{1}{\rho} & \hat{\phi} & \hat{z}\frac{1}{\rho} \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\phi & A_z \end{vmatrix}$$

$$\nabla^2 \psi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \psi}{\partial \phi^2} + \frac{\partial^2 \psi}{\partial z^2}$$

$$\nabla^2 \mathbf{A} = \nabla(\nabla \cdot \mathbf{A}) - \nabla \times \nabla \times \mathbf{A}$$

$$\nabla^2 \mathbf{A} = \hat{\rho}\left(\frac{\partial^2 A_\rho}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_\rho}{\partial \rho} - \frac{A_\rho}{\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 A_\rho}{\partial \phi^2} - \frac{2}{\rho^2}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial^2 A_\rho}{\partial z^2}\right)$$

$$+\hat{\phi}\left(\frac{\partial^2 A_\phi}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_\phi}{\partial \rho} - \frac{A_\phi}{\rho^2} + \frac{1}{\rho^2}\frac{\partial^2 A_\phi}{\partial \phi^2} + \frac{2}{\rho^2}\frac{\partial A_\rho}{\partial \phi} + \frac{\partial^2 A_\phi}{\partial z^2}\right)$$

$$+\hat{z}\left(\frac{\partial^2 A_z}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial A_z}{\partial \rho} + \frac{1}{\rho^2}\frac{\partial^2 A_z}{\partial \phi^2} + \frac{\partial^2 A_z}{\partial z^2}\right)$$

Operadores vectoriales

Coordenadas esféricas

$$\nabla\psi = \hat{r}\frac{\partial\psi}{\partial r} + \hat{\theta}\frac{\partial\psi}{\partial\theta} + \hat{\phi}\frac{1}{r\sin\theta}\frac{\partial\psi}{\partial\phi} \pm$$

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial}{\partial r}r^2 A_r + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}\left(A_\theta\sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial\phi}$$

$$\nabla\times\mathbf{A} = \hat{r}\frac{1}{r\sin\theta}\left(\frac{\partial}{\partial\theta}\left(A_\phi\sin\theta\right) - \frac{\partial A_\theta}{\partial\phi}\right) + \hat{\theta}\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\phi} - \frac{\partial}{\partial r}rA_\phi\right) + \hat{\phi}\frac{1}{r}\left(\frac{\partial}{\partial r}rA_\theta - \frac{\partial A_r}{\partial\theta}\right)$$

$$\nabla^2\psi = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}$$

$$\nabla^2\mathbf{A} = \nabla\left(\nabla\cdot\mathbf{A}\right) - \nabla\times\nabla\times\mathbf{A}$$

$$\nabla^2\mathbf{A} =$$

$$\hat{r}\left(\frac{\partial^2 A_r}{\partial r^2} + \frac{2}{r}\frac{\partial A_r}{\partial r} - \frac{2}{r^2}A_r + \frac{1}{r^2}\frac{\partial^2 A_r}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial A_r}{\partial\theta} + \frac{1}{r^2\sin\theta}\frac{\partial^2 A_r}{\partial\phi^2} - \frac{2}{r^2}\frac{\partial A_\theta}{\partial\theta} - \frac{2\cot\theta}{r^2}A_\theta - \frac{2}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)$$

$$+\hat{\theta}\left(\frac{\partial^2 A_\theta}{\partial r^2} + \frac{2}{r}\frac{\partial A_\theta}{\partial r} - \frac{A_\theta}{r^2\sin^2\theta} + \frac{1}{r^2}\frac{\partial^2 A_\theta}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial A_\theta}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 A_\theta}{\partial\phi^2} + \frac{2}{r^2}\frac{\partial A_r}{\partial\theta} - \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial A_\phi}{\partial\phi}\right)$$

$$+\hat{\phi}\left(\frac{\partial^2 A_\phi}{\partial r^2} + \frac{2}{r}\frac{\partial A_\phi}{\partial r} - \frac{A_\phi}{r^2\sin^2\theta} + \frac{1}{r^2}\frac{\partial^2 A_\phi}{\partial\theta^2} + \frac{\cot\theta}{r^2}\frac{\partial A_\phi}{\partial\theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 A_\phi}{\partial\phi^2} + \frac{2}{r^2\sin\theta}\frac{\partial A_r}{\partial\phi} + \frac{2\cot\theta}{r^2\sin\theta}\frac{\partial A_\theta}{\partial\phi}\right)$$